3-D co-geometry

1. Find the point of intersection of the plane $3x - y + 7z + 8 = 0$ and the line
$x = 4 + 5t, y = -2 + t, z = 4 - t$.
Find also the angle between the line and the plane.

2. The position vectors of points P and Q referred to an origin O are given by $\mathbf{OP} = 3\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{OQ} = 5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$ respectively. Show that the cosine of $\angle POQ$ is equal to $\frac{4}{\sqrt{38}}$.
Hence, or otherwise, find the position vector of M on OQ such that PM is perpendicular to OQ.

3. Given two lines $l_1: x = 1 + 2t,\ y = -2 + 3t,\ z = 2 + 2t$
$l_2: \frac{1-x}{2} = \frac{y-2}{3} = z - 1$
Determine whether these lines are parallel, intersect or skewed.

4. $\pi_1: 2x - y + z + 5 = 0,\ \pi_2: 4x + y + z - 7 = 0$ are two planes.
O is the origin and the point P has coordinates (4,5,2).
   (a) Verify that the vector $\mathbf{r} = -\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ is parallel to both π_1 and π_2.
   (b) Find the vector equation of the plane which passes through P and is perpendicular to both π_1 and π_2.
   (c) Find the coordinates of one point common to π_1 and π_2 and hence, find the Cartesian equation of the line of intersection of π_1 and π_2.

5. The points A, B, C and D have positive vectors, relative to the origin O, given by
$\mathbf{OA} = \mathbf{i} + a\mathbf{j} - \mathbf{k},\ \mathbf{OB} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k},\ \mathbf{OC} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ and $\mathbf{OD} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, where a is a constant.
Given OA is perpendicular to OB,
   (a) find the value of a and **OA**,
   (b) show that **OA** is normal to the plane OBC,
   (c) find an equation of the plane through D parallel to the plane OBC and, hence, find the position vector of the point of intersection of this plane and the line AC.

6. Let the equations of two planes be π_1: $\mathbf{r}\cdot\begin{pmatrix}1\\1\\2\end{pmatrix}=2$ and π_2: $\mathbf{r}\cdot\begin{pmatrix}1\\-3\\4\end{pmatrix}=3$.

(a) Find the acute angle between π_1 and π_2, giving your answer to the nearest $0.1°$.

(b) Determine the length of the projection of the vector $\mathbf{i}+3\mathbf{j}$ to π_1 .

(c) Find the equation of the plane π_3 which is perpendicular to both π_1 and π_2 and passes though the point $P(1,3,-2)$.

7. $\mathrm{A}(1,4,2), \mathrm{B}(3,-1,5), \mathrm{C}(2,6,0), \mathrm{D}(4,3,-1)\mathrm{E}(3,7,3)$ and $\mathrm{F}(4,5,2)$ are six points in three dimensional space. The points A, B and C lie on the plane π_1, whereas the points D, E and F lie on the plane π_2. A straight line L_1 passes through the points E and F.

(a) Determine whether $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ are perpendicular vectors.

(b) Find the Cartesian equation of the plane π_1.

(c) Find the equation of line L_1 in vector form and in parametric form.

(d) Find the coordinates of the point of intersection of the line L_1 and the plane π_1.

(e) Find the Cartesian equation of the straight line L_2 passes through the point $(10,8,9)$ and perpendicular to the plane π_1 .

(f) Find the position vector of the point of intersection between the lines L_1 and L_2 .

(g) Find the acute angle between the plane π_1 and the plane π_2.